The Journal of the Astronautical Sciences, Vol. XXVII, No. 1, pp. 3962, JanuaryMarch, 1979

Anchored Lunar Satellites for Cislunar Transportation and Communication[1]

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Abstract

The concept is examined of anchored lunar satellites, balanced about the collinear libration points Li and L2 of the Earth-moon system and attached to the lunar surface. The design parameters of such satellites are examined by applying the equations of the restricted three-body problem; the material strengths required are within those of available composite materials. Anchored lunar satellites could launch lunar materials throughout cislunar space electrically for 0.75 kilowatt-hour per kilogram, could provide essentially continuous lunar farside communications without stationkeeping propellants, and could supply a lunar base without lunar landing rockets.

Introduction

Since the Apollo lunar landings, the moon has popularly been considered a "dead world," hostile and useless except as a scientific curiosity. Only now are we beginning to realize what a storehouse of raw materials the moon can become. In the development of cislunar space, roughly the disk-shaped region enclosing the libration points of the Earth-moon system, the moon is the most accessible and least fragile source of metals, minerals, soil, and shielding for space colonies. As pointed out by Ehricke [1], when the development of these space habitats begins we will come to realize how fortunate we are in having a "dead world" so conveniently nearby.

In order to use fully the lunar resources, however, a transportation system must be developed to bring thousands of tons of lunar surface material into lunar orbit,

[1]Presented at the European Conference on Space Settlements and Space Industries, London, 20 September 1977. This work is not connected with the author's official dunes at the Flight Dynamics Laboratory.

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to space colonies, and into high Earth orbit. One scheme to alleviate the high cost of rocket launching is that of the electromagnetic launcher proposed by Clarke [2] and developed into a feasible system by O'Neill and his colleagues [3]. A linear motor on the lunar equator would accelerate small pellets of lunar ores into an orbit intersecting the L2 liberation point behind the moon. Here a "mass catcher" would retrieve the pellets and send them via slow space freighter to space colonies for construction of solar power satellites [4].

The purpose of this paper is to investigate an alternative to this system–an alternative which shows the promise of greater efficiency, greater versatility, and eventually greater economy. The technique which will be investigated here is the use of anchored satellites for the launching of lunar materials into cislunar space. After the rocket and the linear accelerator (for airless worlds only), the anchored satellite represents the only known method of launching payloads to planetary escape velocity.

An anchored satellite is simply an extremely long, thin member in tension which is balanced about the stationary altitude and extends to the surface of the planet about which it revolves. Such a planet-to-orbit connection, if successful, would allow unprecedented techniques for launching payloads by using the energy of rotation of the parent body.

The concept of a tower which extends all the way into orbit apparently originated in an 1895 science-fiction story by the Russian rocket pioneer Tsiolkovsky [5]. He realized that a tall tower reaching geostationary orbit would experience a net gravitational force of zero at the top, but he did not see how such a tower could be built. The problem of building the tower was first solved theoretically by a Leningrad engineer, Artsutanov [6], in 1960. Artsutanov was apparently the first to recognize the key requirement that the tower itself must be a satellite in geostationary orbit. This satellite could then be greatly elongated both upward and downward, using the gravity gradient for stabilization, until the lower end of the balanced tower touched the surface at the equator. Artsutanov envisioned a structure large enough to support passenger capsules shuttling between Earth and orbit; he called it a "heavenly funicular." He apparently published no technical papers, however, and his work is known only through the cited article published in *Pravda.*

Unaware of the work of Artsutanov, a group of American oceanographers led by Isaacs [7] independently discovered the concept in 1966 and proposed a much smaller-scale version which they called a "skyhook." They proposed a pair of fine wires which could be alternately raised and lowered by ground-based machines to "walk" payloads into orbit, and performed a static analysis of the wire strength requirements.

The concept for the Earth-to-orbit connection was again discovered independently by this author in 1975 [8]. This proposal was in the form of a large structure called an "orbital tower," and included the first dynamic analysis of the structure. The orbital tower was proposed to recapture the energy of payloads returning from orbit to propel other payloads into orbit. In a later paper [9], the Earth-escape launch capacity of the orbital tower was examined and its limitations were defined in terms of the tower dynamic responses to payload launching forces.

Three independent discoveries in a span of fifteen years indicate that the concept, which may be called an "anchored satellite," is an idea whose time has come. Because the concept is not widely known, the design and capabilities of anchored satellites will be discussed before their applications to the development of cislunar space are investigated. The term "orbital tower," coined by this author, will be used for the general concept of a greatly elongated structure in orbit. If the structure is extended all the way to the parent body, the term "anchored satellite" will also be used.

Characteristics of Orbital Towers

The general features of an orbital tower connected to a planet are shown in Fig. 1. The structure is balanced in tension about the synchronous orbit r_s. The distance to the top for a balanced tower, r_t, is a function of the length of the lower end, which is at the planet's radius r_0. The relation between r_t and r_t is found to be [8]:

$$r_t^3 - (r_0 + 2/r_0)r_t + 2 = 0. \tag{1}$$

In this equation the distances are normalized to $r_s = 1$. For a balanced orbital tower above the Earth, $r_t = 3.56$, corresponding to a radial distance of 150,000 km. Equation (1) is applicable to any single rotating body. For example, Mars has $r_s = 20{,}435$ km and $r_0 = 0.1986\ r_s$. The height of the balanced Martian orbital tower is found from Eq. (1) to be $r_t = 3.0756\ r_s$, or 62,850 km.

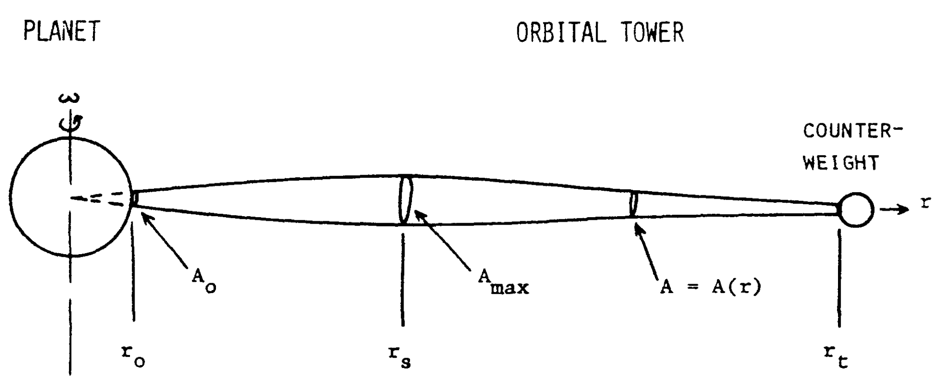

FIG. 1. General Features of an Orbital Tower

Orbital towers are tapered in cross-sectional area to maintain a constant stress due to the tensile force at any point, from a maximum area at r_s to minima at r_0, and r_t:

$$A(r) = A_s exp\left[\frac{r_0^2}{h}\left(\frac{3}{2} - \frac{1}{r} - \frac{r^2}{2}\right)\right], \tag{2}$$

where h is the characteristic height of the building material, the height to which a uniform tower could be built in a one-g gravity field. In this equation h is normalized to the synchronous orbit radius: $h = \sigma/\rho g_0 r_s$. The exponential taper goes to zero area at r = 0 and r = ∞. The tower remains in balance if it is truncated at r_0 and r_t given by Eq. (1). The taper ratio is then defined as $A(r_s)/A(r_0)$.

The orbital tower can be truncated at any point above r_s other than r_t and be maintained in constant stress by attaching a counterweight m_c at the top. This mass is selected so that its upward force on the tower is $\sigma A(r_t)$, where σ is the constant stress limit. The bottom of the tower then experiences an unbalanced upward force of σA_0. If the tower is held in place at the ground, it will have a lifting capacity of σA_0. The size of m_c is a function of its location r_t:

$$m_c \ddot{r_t} = \sigma A(r_t), \tag{3}$$

where $\ddot{r_t}$ is the acceleration at r_t.

Satellite Launching by Orbital Tower

The orbital tower allows travel into space by techniques which are fundamentally different from rockets. By providing a fixed structure to climb, the tower allows almost any kind of mechanical device to be used for propulsion. A motorized capsule with traction wheels to clamp onto the tower would be able to power itself into space. The energy source could be electrical, chemical, nuclear, or solar.

Launching payloads into space by this tower-climbing technique has almost none of the constraints of the rocket. There is no need to expend energy as rapidly as rockets, and thus no need for high thrust or high acceleration. There is no gravity loss, which means that a powered capsule can stop indefinitely at any height without expending energy. It can simply clamp firmly onto the tower until it is prepared to continue.

The tower ascent into orbit can be very efficient; it requires a surprisingly small amount of energy. A payload which climbs to the synchronous orbit has the correct velocity to simply disengage from the tower and remain in orbit. The energy which must be supplied is then just the potential energy difference between the planet's surface and r_s. This is given in terms of the payload mass m and the planet's surface gravity g_0 as:

$$E = \int f(r)dr = mg_0 r_0^2 \int \frac{dr}{r^2} = mg_0 r_0 (1 - r_0/r_s). \tag{4}$$

This result shows that the total energy required to put a payload into geostationary Earth orbit is only 14.8 kWh/kg. At a typical busbar cost of 3 cent per kWh, this is an energy cost of 45 cent/kg into synchronous orbit.

Astonishing as it seems, the orbital tower could conceivably allow payloads to be placed into geostationary orbit for even less than the potential energy difference. A linear induction propulsion system [10] could be used on the tower to propel payloads into orbit electrically. The same system could be used to absorb the energy of descending payloads to generate electrical power. By operating payloads in pairs, the net energy input is only that required to overcome frictional and conversion losses and that required to lift any excess payload into orbit over that being returned.

These figures should completely revise our thinking on the minimum achievable cost of space travel. The Space Shuttle is expected to cost hundreds of dollars per kilogram put into low Earth orbit, and even advanced vehicles are expected to cost tens of dollars per kilogram [11]. Even higher costs are involved for stationary orbit. Because large amounts of carbon and water may need to be exported from Earth to space colonies beyond stationary orbit, it is imperative to bring launch costs down. The orbital tower represents the only known technique for these orders-of-magnitude reductions in Earth-launch costs.

Interplanetary Launching by Orbital Tower

For sending payloads beyond synchronous orbit, the orbital tower has the capability to transfer energy from the Earth's rotation into the orbital motion of payloads located above the synchronous point. Since the tower is balanced about the geostationary point with constant angular velocity, the upper parts have greater than orbital velocity and tend to fly outward. A payload released from the upper part of the tower would be in a higher orbit and could be sent without rocket power to the moon or beyond. This property of the orbital tower means that the energy necessary to reach geostationary orbit is all that is required to send the payload to the moon, Mars, or even solar system escape [8]. This system of launching and recovering space payloads is truly an order of magnitude different from the system of rockets. The Earth orbital tower could launch a continual string of high-mass payloads to escape velocity for no additional energy over that needed to attain geostationary orbit [9].

It is possible to imagine an interplanetary transportation system between the Earth and Mars, for example, consisting of an orbital tower attached to each body. Electrical propulsion systems could send payloads from the ground to synchronous orbit launch platforms, from which they would be released at the proper times to rendezvous with the other planet. By mid-course and terminal guidance, the payloads could match velocities with the receiving tower at a point above the synchronous orbit. They could then climb down the tower under electrical power to the surface of the planet, eliminating the need for heat shields.

Construction Requirements

There are serious obstacles to the successful construction of an Earth orbital tower; the foremost problem is the required strength-to-weight ratio of the building material. The only materials which promise the required strengths are the whiskers of perfect-crystals which have been made on a laboratory scale [12]. These whiskers, a fraction of a centimeter long and a few microns in diameter, show nearly the theoretical single-crystal tensile strength. If whiskers of alumina, graphite, silicon carbide, or boron could be successfully produced and bonded in a suitable matrix, they could be used as the Earth orbital tower material. At present, however, there is no practical building material available for an efficient, low-mass Earth tower.

In spite of the difficulties, the potential of the orbital tower for efficient, ecologically harmless launching of extremely large payloads makes it worthwhile to pursue the concept. In addition, the strength requirement is significantly less for construction of orbital towers about bodies of lower mass than the Earth, such as the moon.

Anchored Lunar Satellites

The orbital towers, or anchored satellites, discussed so far have been applied to the stationary orbit of a single planet. A more complex situation is the construction of an anchored satellite about the unstable libration points of a planet-moon system. In this case the satellite is constructed from the libration points L1 or L2 to the surface of the moon, which must have a captured rotation (one side always facing the planet). Isaacs et al. [7] suggested the construction of a connection between L2 and our moon's farside, but did not analyze this situation. The analysis which follows examines the general characteristics of such anchored lunar satellites, assesses their difficulties of construction and their launch capabilities, and discusses possible applications in the development of cislunar space.

The analysis of anchored lunar satellites is an application of the restricted three-body problem. In this case the primary bodies are the Earth and the moon, assumed to revolve in circular orbits about their barycenter, as shown in Fig. 2. Using in general the notation of Szebehely [13], the barycenter is the origin of coordinates, the x-y plane is the orbital plane, Earth is of mass $1-\mu$ at (μ, 0) and the moon is of mass μ at (μ-1,0). The quantity μ is the ratio of the moon's mass to the total mass, in this rotating coordinate system, the Earth-moon distance and total mass are normalized to one and the angular velocity of the moon is also set

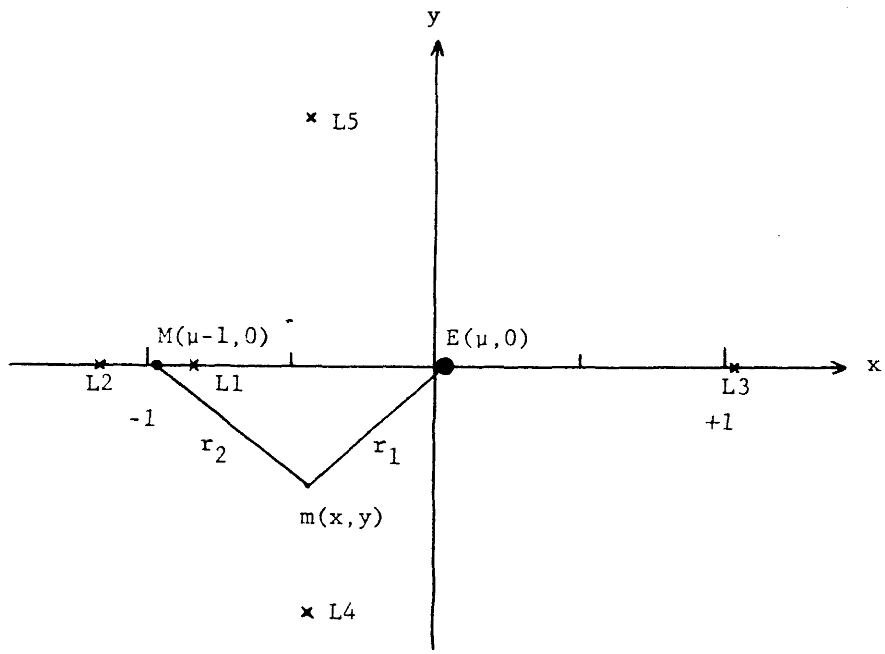

FIG. 2. Notation for Earth-Moon-Spacecraft Problem

to one. With a unit distance of 384,410 km, the normalized time is 104.362 hours, the unit velocity is 1023.17 m/s, and unit acceleration is 0.00273 m/s^2. The value of μ is taken to be 1/82.30.

The third body shown in Fig. 2 is located at distances r_1 and r_2 from the Earth and the moon, respectively, and is of such small mass that it does not affect their motions. The equilibrium points discovered by Lagrange are also shown in Fig. 2. The triangular points L4 and L5 are stable positions for third bodies; the collinear points L1, L2, and L3 are unstable equilibrium points.

We first assume that an anchored satellite is to be built about L2, which is at a mean distance behind the moon of 64,517 km. Referring to Fig. 3, the weight of a slice of the satellite of length dx is:

$$dW = \rho A \left(\frac{GM_e}{r_1^2} + \frac{GM_m}{r_2^2} - \omega^2 x \right) dx, \tag{5}$$

where ρ and A are the tower density and cross-sectional area, r_1 = x + μ, r_2 = x - 1 + μ, and ω is the angular velocity of the moon,

$$\omega^2 = \left(\frac{2\pi}{T} \right)^2 = \frac{G(M_e + M_m)}{a^3}$$

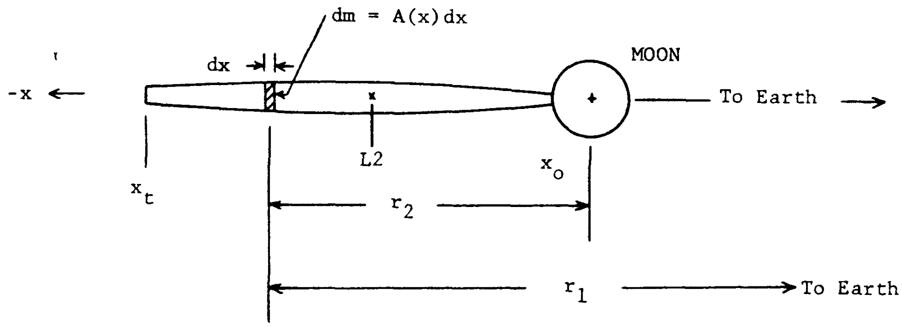

FIG. 3. Notation for Anchored Lunar Satellite Problem

where T and a are the period and semi-major axis of the moon's orbit. Using the normalization that a $= 1$, $\omega = 1$, Eq. (5) reduces to

$$W = \int_{x_0}^{x_t} \rho A \left(\frac{1-\mu}{r_1^2} + \frac{\mu}{r_2^2} - x \right) dx = 0 \tag{6}$$

for a balanced tower, where x_0 is at the moon's surface and x_t is the unknown location of the tower top. Integration gives:

$$\frac{x_t^2}{2} + \frac{1-\mu}{x_t+\mu} + \frac{\mu}{x_t+\mu-1} = \frac{x_0^2}{2} + \frac{1-\mu}{x_0+\mu} + \frac{\mu}{x_0+\mu-1}. \tag{7}$$

After some algebraic manipulation a quartic equation in x_t is derived which can be solved numerically. The result is that $x_t = 2.36$, and the length of the satellite is 525,724 km. This means that the top of the balanced satellite must be extremely distant to counterbalance the weight of the lower part of the satellite. A similar analysis of the balanced anchored satellite about L1 shows it to have a length of 291,901 km.

Because these structures must support loads which vary along their lengths, their cross-sectional areas may be optimized by sizing them for a constant stress. Using Eq. (5) for the differential weight, dW, of the L2 satellite,

$$dW = \rho A \left(\frac{GM_e}{r_1^2} + \frac{GM_m}{r_2^2} - \omega^2 x \right) dx = \sigma dA,$$

where σ is the constant stress. Re-arranging gives

$$\frac{dA}{a(x)} = \frac{\rho}{\sigma} G(M_e + M_m) \left(\frac{1-\mu}{(x+\mu)^2} + \frac{\mu}{(x+\mu-1)^2} - \frac{x}{a^3} \right) dx.$$

Integrating and setting $A(x = L_i) = A_{max}$ gives the cross-sectional area as a function of the distance:

$$A(x) = A_{max} exp \left[\frac{2.7791x10^{-4}}{h} \left(\frac{1-\mu}{\mu - x} \mp \frac{\mu}{\mu - x - 1} - \frac{x^2}{2} - K_i \right) \right], \tag{8}$$

where

$$K_i = \left[\frac{1-\mu}{\mu - x_{Li}} \mp \frac{\mu}{\mu - x_{Li} - 1} - \frac{x_{Li}^2}{2} \right].$$

The upper sign refers to the L1 tower ($i = 1$) and the lower sign to the L2 tower ($i = 2$). The taper required is seen to be an exponential function of the strength-to-weight ratio, very similar to the result for the Earth-anchored satellite [8]. These balanced anchored lunar satellites are shown to scale (with exaggerated diameters) and compared to the balanced Earth anchored satellite in Fig. 4.

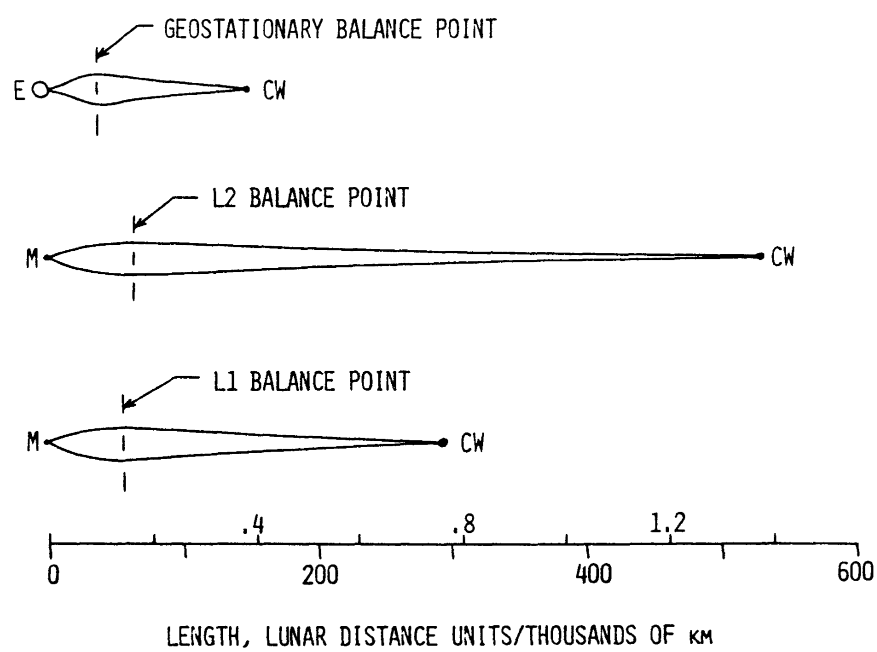

FIG. 4. Anchored Lunar Satellites Compared to the Anchored Earth Satellite

Design Requirements for Anchored Lunar Satellites

The taper ratios, A_{max}/A_0, for the L1 and L2 anchored satellites are shown in Fig. 5 as functions of the characteristic height of the building material. The characteristic height is the height that a uniform column of the material could attain in a gravity field of g_e, the value at the surface of the Earth, without exceeding the stress limit at the base: $h = \sigma/\rho g_e$. The taper ratio required for the anchored Earth satellite is also shown in Fig. 5 for comparison.

The anchored lunar satellites are seen to be far easier to construct than the anchored Earth satellite. They could be made from existing engineering materials such as those shown in Table I. Data for these composite materials have been taken from the Advanced Composite Design Guide [14]. The graphite/epoxy composite could be used to construct anchored lunar satellites with taper ratios of less than thirty, but there is some question about the availability of carbon on the moon. In order to use this material, an outside source of carbon might be required, such as

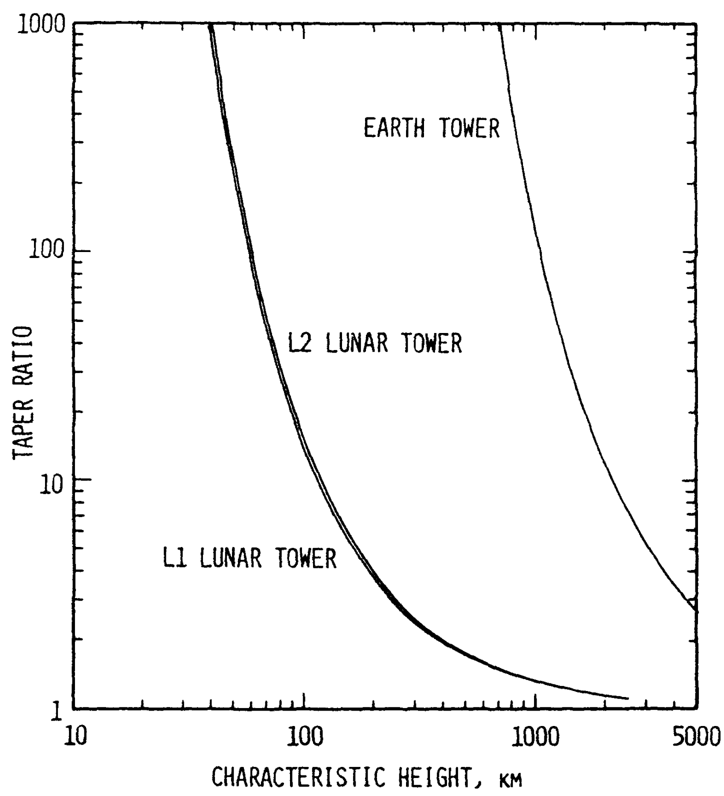

FIG. 5. Taper Ratios Required for Anchored Lunar Satellites

TABLE I. Candidate Materials and Properties for Anchored Satellites

	Design Strength GN/m^2	Density kg/m^3	Characteristic height, km
Graphite/Epoxy	1.24	1550	81.6
Boron/Epoxy	1.32	2007	67.3
Kevlar 49	0.703	1356	52.9
Boron/Alumninum	1.10	2713	41.5

carbonaceous chondrites. Aluminum and boron are far more common [1], but the boron/epoxy would require a taper ratio of nearly 100 and the boron/aluminum composite even more.

The relative amounts of material required to build the L1 and the L2 anchored lunar satellites are shown in Figs. 6 and 7, respectively. The satellite mass per unit base area is given as a function of the satellite length for a taper ratio of 30. These figures also show the mass of the counterweight required to balance the satellite as a function of the satellite length. As an example, an L1 satellite with a base area of $10^{-4} m^2$ constructed of graphite/epoxy would total 5.31 X 10^8 kg. It would have a lifting capacity of 74,400 kg at the base, or 1,316,000 kg if this mass were spread uniformly between the lunar surface and L1. If this material were carried upward at an average velocity of 375 m/s, then in one year 2.77 X 10^8 kg could be carried to L1. From this point it could be launched without rockets to many points in cislunar space.

The mass and carrying capacity (kilograms of payload per kilogram of satellite mass) are shown in Fig. 8 as functions of the characteristic height of the material. Note that the Ll anchored satellite is lighter than the L2 satellite for the same carrying capacity. Note also that increased characteristic height dramatically increases the payload ratioa change from 75 to 100 km, for example, more than doubles the payload ratio.

A different measure of the carrying capacity of the anchored satellite is the time required to carry its own mass into orbit. Table II shows these duplication times for various characteristic heights of the building material for the L1 satellite. Doubling the characteristic height from 60 to 120 km decreases the duplication time to about one fourteenth. This result again shows the importance of using the strongest possible material.

Construction Technique

To begin the construction of these satellites, an initial thin strand of material would be carried by rocket to L1 or L2, where it would be uncoiled and extended to the surface from a manned construction module. The upper end could be terminated far short of the balance point x_t given by Eq. (7) by attaching a large

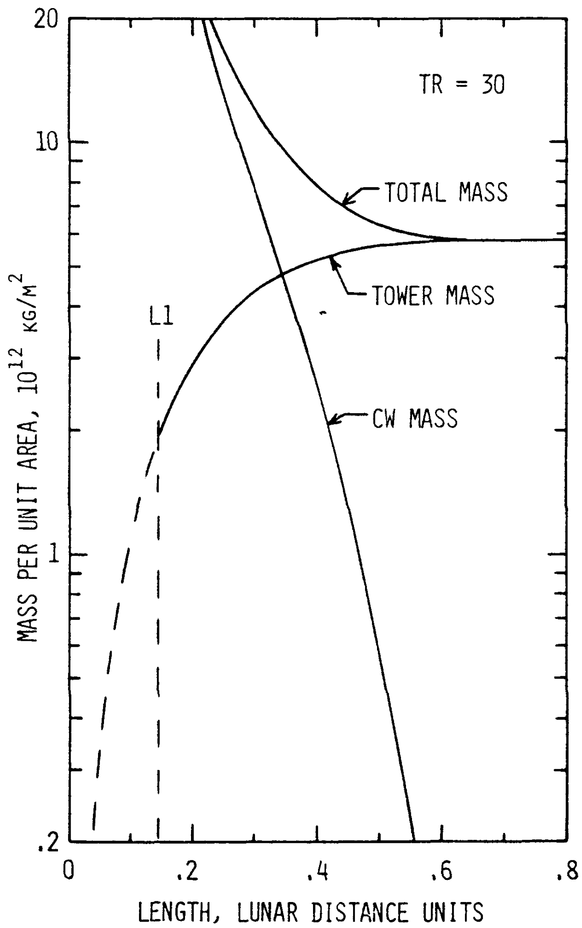

FIG. 6. L1 Lunar Satellite Mass and Counterweight

counterweight of mass m_c. The use of a counterweight would add to the total mass required, but it would reduce considerably the mass of high-strength material needed. The counterweight could be almost any kind of inert mass, such as excess lunar surface slag from the processing of the high-strength building material or the remains of a carbonaceous chondrite used for its carbon.

Once the initial strand was attached to the surface and put into tension by the counterweight, it could be used to support self-powered mechanical climbers. These devices could then carry additional strands of material to build up the satellite to its final size. The counterweight would be the proper mass to cause a net upward force on the base of the satellite of σA_0 newtons. This force is then the net lifting capacity at the base.

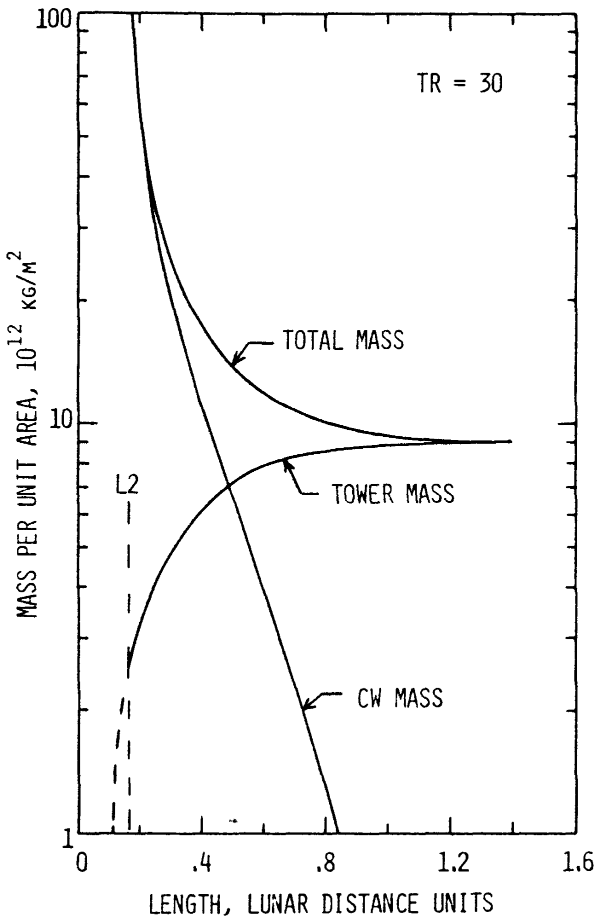

FIG. 7. L2 Lunar Satellite Mass and Counterweight

Because of the instability of L1 and L2, the anchored lunar satellite is not fail-safe. This contrasts with the Earth anchored satellite, which would continue in stable orbit if it were severed near the base and then jettisoned its counterweight. A break in the lunar satellite, however, would tend to make the upper part fly off into space or crash to the surface even if the counterweight were released. To prevent this from happening, an emergency stabilization system would be required. Such a system could take the form of a powered module located near the balance point and capable of lengthening or shortening the cable. Farquhar [15] has analyzed the stability of cable-connected masses about Ll and L2 and found that this configuration could be stabilized by controlling the length of the cable. In case of a break in the satellite, part of the ballast mass could be released to bring the

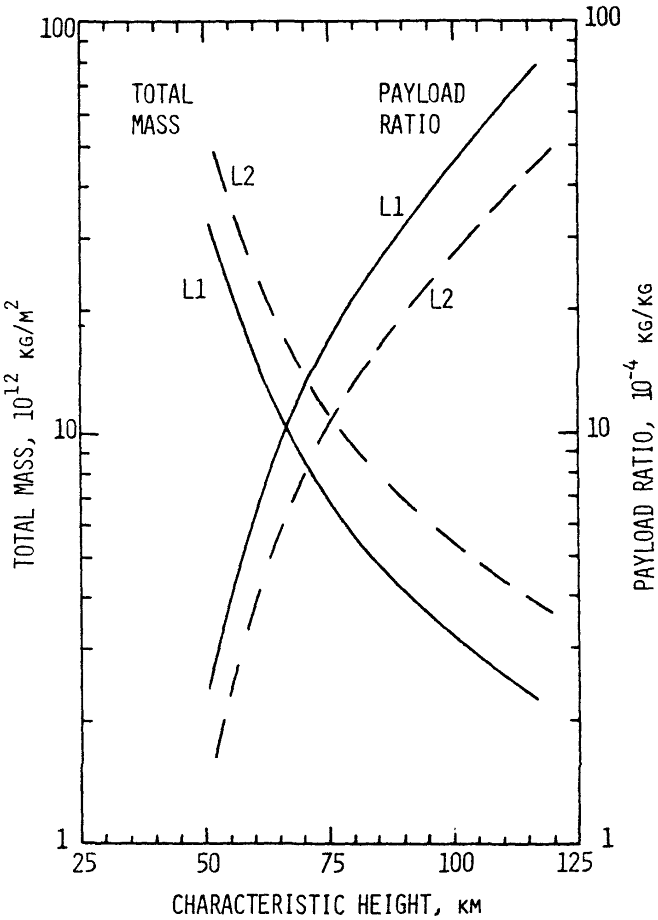

FIG. 8. Mass and Payload Capacities for Lunar Anchored Satellites

TABLE II. Duplication Times for Anchored L1 Lunar Satellites

Characteristic height, km	Payload ratio	Duplication time years
60	6.2 x 10^{-4}	7.67
80	21.3	2.23
100	45.4	1.05
120	85.7	0.55

remainder of the satellite into balance; the stabilization module could then maintain the satellite in position until a repair could be made. The repair could take the form of extra strands of material lowered along the satellite to the lunar surface.

Applications of Anchored Lunar Satellites

Anchored lunar satellites can be used in the same manner as the Earth orbital tower to extract energy from the moon's motion and use it to launch orbital or escape payloads. The potential energy required to lift a mass from the moon's surface to L2, for example, can be found by integrating Eq. (5):

$$E = f dx = m\omega^2 \int_{x0}^{x(L2)} \left(\frac{1-\mu}{(x+1-\mu)^2} + \frac{\mu}{(x+\mu-1)^2} - \frac{x}{a^3} \right) dx \tag{9}$$

This amount of energy is 0.749 kilowatt-hours per kilogram. Once the payload is supplied with this energy by lifting it to the libration point, it can be allowed to slide to higher points of the satellite without additional energy and then be released to travel in cislunar space.

The dynamics of a body released from the lunar tower are very complex, because the gravity effects of both the moon and the Earth must be taken into account. The motion must be analyzed as a restricted three-body problem, which has received extensive analytical treatment. Using the notation of Szebehely, Fig. 2, the planar equations of motion of a small body in the vicinity of two massive bodies in circular orbits are:

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} = x - \frac{(1-\mu)(x-\mu)}{r_1^3} - \frac{\mu(x+1-\mu)}{r_2^3}, \tag{10}$$

and

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} = y \left[k - \frac{1-\mu}{r_1^3} - \frac{\mu}{r_2^3} \right]. \tag{11}$$

Here the distances to the primaries are defined as:

$$r_1^2 = (x-\mu)^2 + y^2, r_2^2 = (x-\mu+1)^2 + y^2, \tag{12}$$

and $\Omega(x, y)$ is a potential function from which the Jacobian constant C is defined:

$$\Omega(x, y) = \frac{x^2+y^2}{2} + \frac{1-\mu}{r_1} + \frac{\mu}{r_2} + \frac{\mu(1-\mu)}{2}, \tag{13}$$

and

$$C = 2\Omega - \dot{x}^2 - \dot{y}^2. \tag{14}$$

These equations were programmed on a digital computer in order to find the trajectories of small masses released from various points on the anchored lunar satellites. Examples of these trajectories are shown in the next two illustrations. Figure 9 shows the orbit of a body released from near L2 with a small velocity toward the moon. This body performs a few orbits of the moon, then escapes through Ll into an elliptical orbit around the Earth. In contrast to this, Fig. 10 shows the trajectory of a body released from near L2 with a small velocity away from the moon. The result is a large elliptical orbit about the Earth and moon.

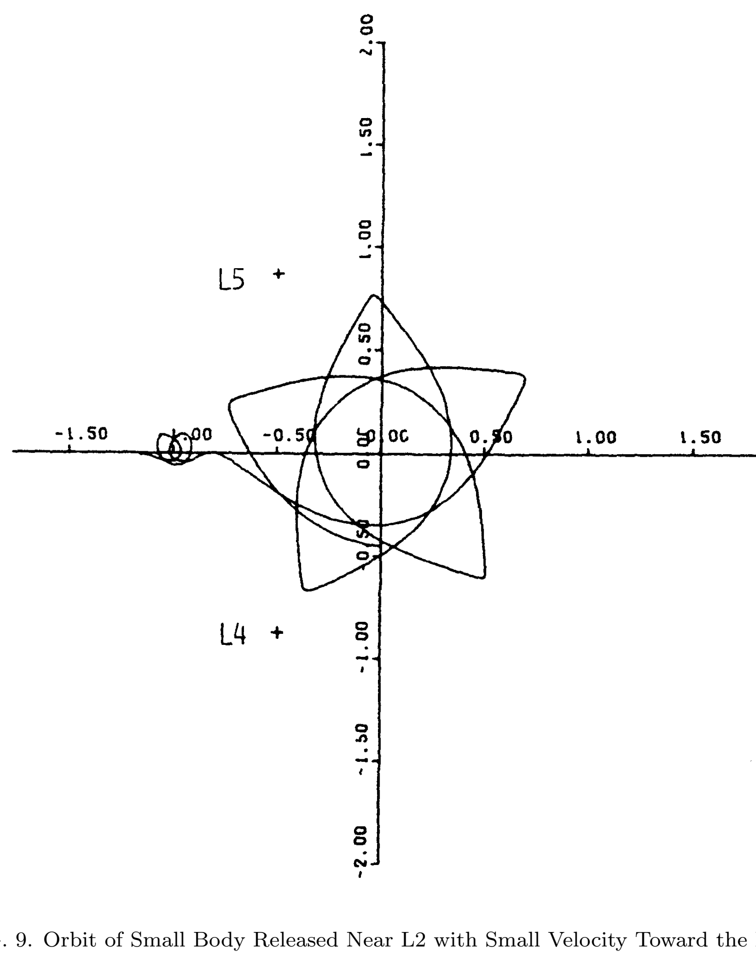

FIG. 9. Orbit of Small Body Released Near L2 with Small Velocity Toward the Moon

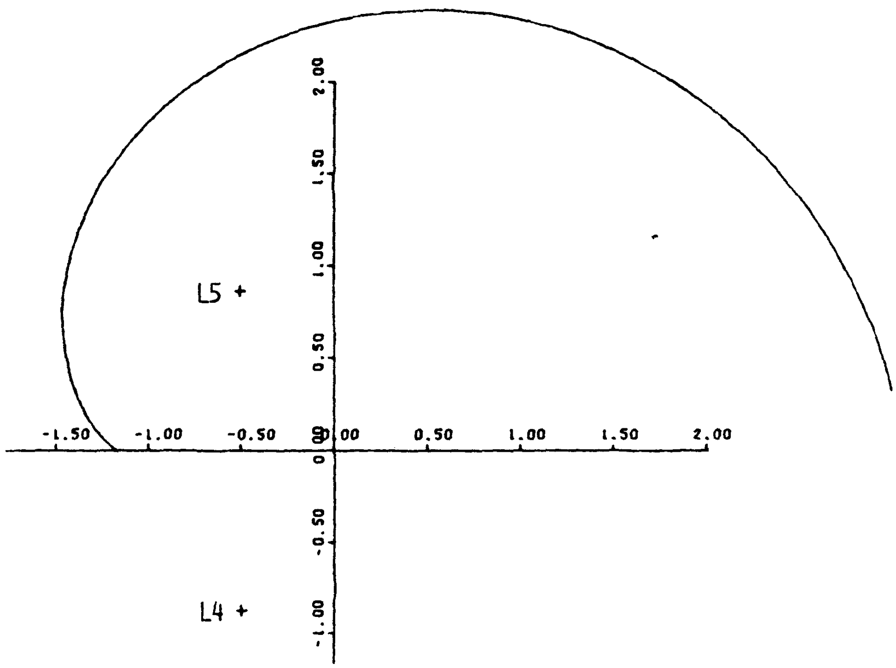

FIG. 10. Orbit of Small Body Released Near L2 with Small Velocity Away from the Moon

This computer run was stopped when r_1 reached three times the lunar distance. The difference in velocity between the two trajectories of Figs. 9 and 10 was just 0.02 units, or 20 m/s.

These examples show that a small change in the velocity of a body released from the L2 lunar satellite can drastically change the area accessible to it. A large part of cislunar space is therefore accessible from near L1 and L2 with very small velocity requirements.

The general features of such orbits can be understood by a plot of curves of zero velocity taken from Szebehely and shown in Fig. 11. This diagram shows the curves of constant C as a function of x and y for bodies of zero initial velocity with respect to the rotating coordinate system. These curves are not orbits; rather, they represent the boundaries of the region in which travel is possible by bodies of a given energy. These curves are symmetrical about the x axis, so only the upper half of the x-y plane is shown. The lower part of Fig. 11 shows the region about the moon in greater detail.

In this diagram, the first important point is that the highest values of the Jacobian constant are represented by the Earth and the moon; at these "point masses" C increases without bound. A body near the Earth or moon with a large

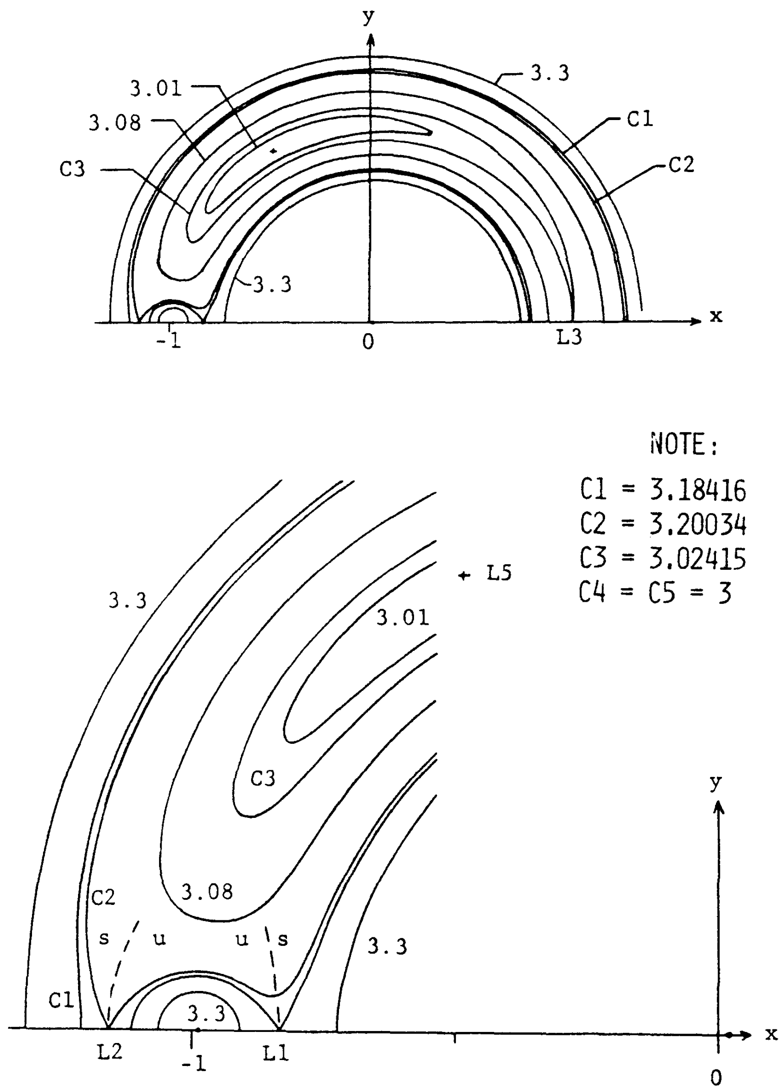

FIG. 11. Curves of Zero Velocity in the Earth-Moon System

value of C (low velocity) is constrained to remain within the curve about that body with that value of C. Secondly, notice that the stable libration points L4 and L5 have the lowest value of $C(C = 3)$. These positions represent extreme points on the energy diagram and require the most energy to be accessible to a spacecraft.

The third important feature of Fig. 11 is that the unstable libration points are represented by saddle surfaces. Along the x axis the value of C increases as we move away from L1, L2, and L3. Conversely, in the y direction the value of C decreases away from these libration points. What this means practically is that a small change in initial energy near one of these points results in radically different orbits.

Figure 12 defines the areas accessible to masses released from the L1 and L2 points on the anchored lunar satellites. The upper part of the figure shows the region accessible to the L2 release point by shading. The lower part shows the L1 accessible region shaded. The difference is that large orbits about both Earth and moon are accessible from L2 but not from L1. This extra capability from L2 can be represented by the velocity difference of $\sqrt{C(Ll) - C(L2)}$, or 130 m/s.

This diagram shows that a large volume of cislunar space is accessible from L1 or L2, including Earth orbits and lunar orbits. However, the L4 and L5 points are both excluded. This difficulty in reaching L4 and L5 has led Heppenheirner and Kaplan [41 to propose that space colonies be located in an orbit about the Earth with a period of half a month, approximately as shown in Fig. 9. This orbit can

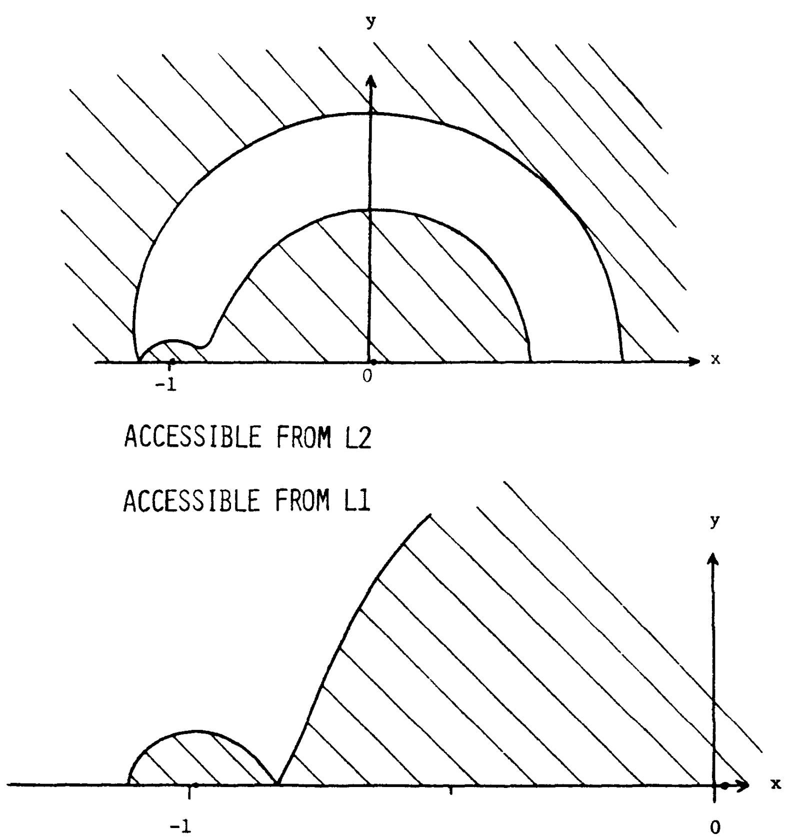

FIG. 12. Areas Accessible to Masses Released from the L1 and L2 Satellites

be reached easily from L2 or L1. In contrast, L5 requires a velocity change of 440 m/s from L2, as can be seen by comparing the difference in the Jacobian constant between them of $\sqrt{C(L2) - C(L5)} = 0.430$ units.

We can now summarize how the L1 lunar satellite could be used in the commerce of cislunar space. Material would be mined and refined at the base of the satellite, on the equator in the center of the near side. A manned station at the libration point would receive the lunar material and send it to its destination. Allowing it to slide down the tower toward the moon and then releasing it brings various lunar equatorial orbits in reach. Releasing with small velocities near L1 puts Earth orbit into reach; such launches could be used to send material to rendezvous with the upper part of the Earth orbital tower. By using an electrical propulsion system on the upper part of the Earth orbital tower, these payloads could be Sent to the Earth's surface or synchronous Earth orbit without rockets.

This cislunar transportation system can work in reverse, with the L1 station receiving goods from Earth or the space colonies and sending them to the base of the lunar satellite to supply the lunar base. The system could also function essentially identically by using the L2 satellite; the higher energy orbits about Earth-moon would then be accessible. The L1 satellite has the advantages of being slightly easier to build and having significantly greater payload capacity. It would also have the advantage of being constructed on the near side of the moon, being in continuous view from the Earth.

The Anchored Lunar Halo Satellite

The concept of the L2 anchored satellite lends itself to a communications link to the moon's farside by using the "halo" orbits shown in Fig. 13. Farquhar [16] originally proposed stationing a relay satellite in a quasi-periodic halo orbit about L2 in order to provide continuous communication between the Earth and the

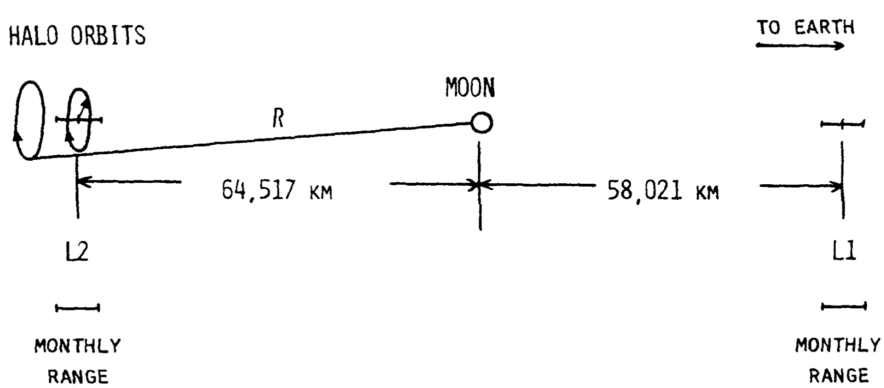

FIG. 13. Lunar Halo Orbit for Farside Communication

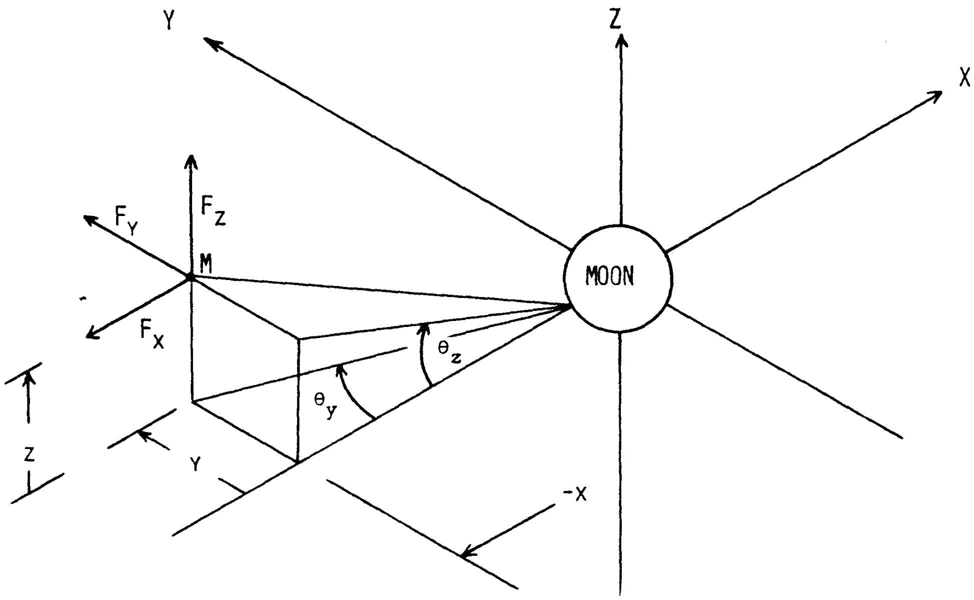

FIG. 14. Notation for Anchored Lunar Halo Satellite

moon's farside. The halo orbit could be large enough to keep the satellite in continuous view of the Earth and also of a second satellite at L1. With accurate knowledge of the halo satellite's position, an active control system could maintain a stable orbit with small amounts of reaction propellant or with a solar sail, as suggested by Colombo [17].

The anchored L2 lunar satellite could also function as such a communication link, using the cable as a completely passive position control system. The concept is shown in Fig. 14 as a mass m in a halo orbit past L2 and attached to the center of the lunar farside by a tapered cable of length R. The cable is sized to maintain the mass m in balance by its tensile force, making the mass equivalent to the counterweight given in Fig. 7. This system would act as a spherical pendulum free to swing in the y and z directions.

The motion of such an anchored satellite in three dimensions was examined under the assumption that the mass of the cable can be neglected compared to the mass of the satellite. The equations of motion in terms of the two angles θ_y and θ_z are:

$$\ddot{\theta}_y = \frac{F_x}{mR} sin\theta_y + \frac{F_y}{mR} cos\theta_y$$
$$\ddot{\theta}_z = \frac{F_x}{mR} sin\theta_z + \frac{F_z}{mR} cos\theta_z, \tag{15}$$

where the forces $F_y = m\ddot{y}$ and $F_z = m\ddot{z}$ are given by Eqs. 10 and 11, and the distances are given by Eqs. (12).

These equations were first examined for the region of static stability. Using Fig. 14 as a guide, the anchored halo satellite is stable if the net force on the mass is in the negative x direction, opposing the cable tension. The stability boundary in the $x - y$ plane is thus:

$$F_x sin\theta_y + F_y cos\theta_y = 0,$$

in which the forces are evaluated with all velocities equal to zero. The resulting stability boundaries for both the L1 and L2 satellites are shown dashed in Fig. 11. The side of each boundary marked "s" represents the stable region and the side

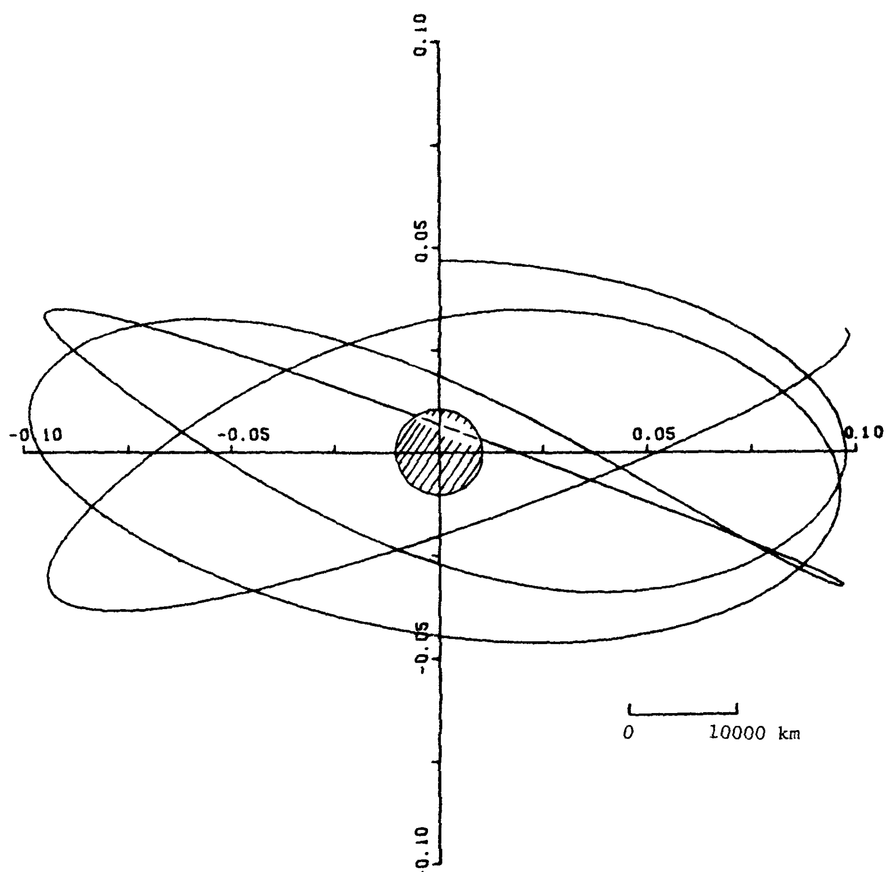

FIG. 15. Example Path of Anchored Lunar Halo Satellite Seen from L1

marked "u" is the unstable region. In order for the mass to maintain a positive tension in the cable, the minimum cable length is the distance from the surface to L2 when the moon is at apogee, which is 68,056 km.

The nonlinear spherical-pendulum Eqs. (15) were programmed for a digital computer. The motion of the mass was found to be in general a complex Lissajous pattern, as shown in Fig. 15. These Lissajous paths occasionally cross the origin, resulting in communication blackouts by lunar occultations. The typical trajectory of Fig. 15, however, would result in communication loss during eclipses of only 11 hours in 65 days. The shaded circle shows the area in which the satellite would be hidden from L1.

Putting the anchored lunar halo satellite at near the minimum altitude for stability, say 70,000 km, means that the cable can be very small because of the nearly zero force on the mass. At this distance the mass of the satellite can be thirty times the mass of the cable. For a satellite mass of a few thousand kilograms, an extremely fine wire would suffice, with a total mass on the order of a few hundred kilograms.

Conclusions

The concept of anchored lunar satellites balanced about the collinear libration points L1 and L2 of the Earth-moon system has been examined. Such satellites would be far easier to construct than the corresponding Earth-anchored satellite in geostationary orbit. The anchored lunar satellites could be used to supply lunar materials to space colonies located at a variety of points in cislunar space without rocket power. The L4 and L5 points, however, cannot be reached without rocket thrust. The L1 satellite offers easier construction and higher lifting capacity. The L2 satellite offers advantages in communication with the lunar farside. The use of such anchored satellites for supplying lunar materials to space colonies could reduce transportation costs to nearly the theoretical minimum. Anchored satellites also provide a means of supplying a lunar base from Earth or space colonies without lunar landing rockets.

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